Published by Institute of Physics Publishing for SISSA

Received: *August 3, 2005*
Accepted: *September 10, 2005*
Published: *October 4, 2005*

Non relativistic Dp branes

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Abstract: We construct a kappa-symmetric and diffeomorphism-invariant non-relativistic Dp-brane action as a non-relativistic limit of a relativistic Dp-brane action in flat space. In a suitable gauge the world-volume theory is given by a supersymmetric free field theory in flat spacetime in $p+1$ dimensions of bosons, fermions and gauge fields.

Keywords: AdS-CFT and dS-CFT Correspondence, D-branes.

Contents

1. Introduction

Non-relativistic string theory [1, 2] is a consistent sector of string theory, whose world-sheet conformal field theory description has the appropriate galilean symmetry [3]. Non-relativistic superstrings and non-relativistic superbranes [4, 5] are obtained as a certain decoupling limit of the full relativistic theory. The basic idea behind the decoupling limit is to take a particular non-relativistic limit in such a way that the light states satisfy a galilean-invariant dispersion relation, while the rest decouple. For the case of strings, this can be accomplished by considering wound strings in the presence of a background B-field and tuning the B-field so that the energy coming from the B-field cancels the tension of the string. In flat space, once kappa symmetry and diffeomorphism invariance are fixed, non-relativistic strings are described by a free field theory in flat space. In $\mathrm{AdS}_5 \times S^5$ [6], the world-sheet theory reduces to a supersymmetric free field theory in AdS_2.

In this paper we study the non-relativistic limit of non-perturbative supersymmetric objects of string theory. We study non-relativistic supersymmetric Dp branes in flat spacetime. The point of departure is to consider the world-volume kappa invariant action of a relativistic Dp brane in flat spacetime [7–10]. Since the Dp branes are charged under the RR forms, we also consider its coupling to a closed $p+1$ RR form, C_{p+1}. In this way we can find a limit where the tension of the wound Dp brane is cancelled by the coupling to the C_{p+1} field. Only states with positive charge remain light in the limit, while the non-positively charged states become heavy. We obtain a world-volume kappa symmetric action of a non-relativistic Dp brane. When kappa symmetry [11] and diffeomorphisms are fixed, the non-relativistic Dp-brane action is described by a supersymmetric *free* field theory in flat spacetime in $p+1$ dimensions of bosons, fermions and gauge fields. This is the main result.

JHEP10(2005)007

The paper is organized as follows. In section 2, we summarize the basic properties of kappa-symmetric relativistic Dp-brane actions in flat space. In section 3, we consider the non-relativistic limit of relativistic Dp branes. The supersymmetry and kappa transformations are discussed in section 3.1. It is shown there how after the gauge fixing these transformations give rise to a rigid supersymmetric vector multiplet with the usual supersymmetry algebra. In section 3.2, we will specify to the case of a D string. While in other cases the Wess-Zumino (WZ) term is given implicitly as a $(p+2)$-form over an embedding manifold, in this case the form of the WZ term is simple and we give it explicitly. We finish by some conclusions and an appendix with conventions.

2. Relativistic Dp branes

The action for a relativistic Dp brane propagating in flat space[1] is [7−10]

$$\begin{aligned} S &= -T_p \int \mathrm{d}^{p+1}\sigma \sqrt{-\det\left(G_{ij}+\mathcal{F}_{ij}\right)} + T_p \int \Omega_{p+1} \\ &= -T_p \int \left[\mathcal{L}_{\mathrm{NG}} - \mathcal{L}_{\mathrm{WZ}}\right] = -T_p \int \mathcal{L}_{\mathrm{GS}} \,. \end{aligned} \tag{2.1}$$

G_{ij} is the induced metric constructed from the supertranslation invariant 1-form

$$\Pi^m = \mathrm{d}X^m + i\bar{\theta}\Gamma^m \mathrm{d}\theta \,. \tag{2.2}$$

$\mathcal{F}_{ij}$ is constructed from the two form $\mathcal{F} = 2\pi\alpha' F - b$, which is written in terms of the field strength of the Born-Infeld (BI) field, A, and of the pullback of the fermionic components of the B field in superspace. $\mathcal{L}_{\mathrm{WZ}} = \Omega_{p+1}$ is the WZ term. Since the expression is complicated[2], it is useful to introduce a $(p+2)$-form h_{p+2} such that

$$h_{p+2} = \mathrm{d}\Omega_{p+1} \,. \tag{2.3}$$

For type-IIA Dp branes (p even), the forms are given by

$$\begin{aligned} b &= -i\bar{\theta}\Gamma_{11}\Gamma_m \mathrm{d}\theta \left(\Pi^m - \frac{i}{2}\bar{\theta}\Gamma^m \mathrm{d}\theta\right), \\ h_{p+2} &= (-)^n i \mathrm{d}\bar{\theta}\mathcal{T}_p \mathrm{d}\theta \,, \qquad p = 2n \,, \end{aligned} \tag{2.4}$$

where $\mathcal{T}_p$ is a p form. To define it, we introduce the formal sum of differential forms

$$\mathcal{T}_A = \sum_{p=\mathrm{even}} \mathcal{T}_p = \mathrm{e}^{\mathcal{F}} C_A \,, \tag{2.5}$$

where

$$C_A = \Gamma_{11} + \frac{1}{2!}\psi^2 + \frac{1}{4!}\Gamma_{11}\psi^4 + \frac{1}{6!}\psi^6 + \cdots \,. \tag{2.6}$$

[1] We are using conventions close to these of [7, 8], except that the exterior derivative commutes with θ, and our spinor conventions imply that their $\bar{\theta}$ is $-i\bar{\theta}$ for us. See the appendix for more details.

[2] The explicit form of the WZ term is given in [12, 13].

and

$$\psi = \Pi^m \Gamma_m \,. \tag{2.7}$$

The Dp-brane action (2.1) is invariant under the supersymmetry transformations

$$\begin{aligned}
\delta_\epsilon \theta &= \epsilon, \qquad \delta_\epsilon X^m = -i\bar\epsilon\Gamma^m\theta\,, \\
\delta_\epsilon(2\pi\alpha' A) &= -i\bar\epsilon\Gamma_{11}\Gamma_m\theta \mathrm{d}X^m + \frac{1}{6}\left(\bar\epsilon\Gamma_{11}\Gamma_m\theta\bar\theta\Gamma^m\mathrm{d}\theta + \bar\epsilon\Gamma_m\theta\bar\theta\Gamma_{11}\Gamma^m\mathrm{d}\theta\right),
\end{aligned} \tag{2.8}$$

and under the kappa transformations

$$\begin{aligned}
\delta_\kappa\bar\theta &= \frac{1}{2}\bar\kappa\left[1+(-)^n\Gamma_\kappa\right], \qquad \delta_\kappa X^m = -i\bar\theta\Gamma^m\delta_\kappa\theta\,, \\
\delta_\kappa(2\pi\alpha' A) &= +i\delta_\kappa\bar\theta\Gamma_{11}\Gamma_m\theta\Pi^m + \frac{1}{2}\delta_\kappa\bar\theta\Gamma_{11}\Gamma_m\theta\bar\theta\Gamma^m\mathrm{d}\theta - \frac{1}{2}\delta_\kappa\bar\theta\Gamma_m\theta\bar\theta\Gamma_{11}\Gamma^m\mathrm{d}\theta\,,
\end{aligned} \tag{2.9}$$

where

$$\Gamma_\kappa = \frac{1}{(p+1)!}\frac{\varepsilon^{i_0\ldots i_p}}{\sqrt{-\det(G+\mathcal{F})}}\left(\rho_{p+1}\right)_{i_0\ldots i_p} \,. \tag{2.10}$$

The $(p+1)$-form ρ_{p+1} is defined [7, 8] by the formal sum

$$\rho_A = \sum_{p=\mathrm{even}} \rho_{p+1} = \mathrm{e}^{\mathcal{F}} S_A\,, \tag{2.11}$$

where

$$S_A = \Gamma_{11}\psi + \frac{1}{3!}\psi^3 + \frac{1}{5!}\Gamma_{11}\psi^5 + \frac{1}{7!}\psi^7 + \cdots \,. \tag{2.12}$$

For the type-IIB Dp branes (p odd) we have

$$\begin{aligned}
b &= -i\bar\theta\Gamma_m\tau_3\mathrm{d}\theta\left(\Pi^m - \frac{i}{2}\bar\theta\Gamma^m\mathrm{d}\theta\right), \\
h_{p+2} &= i\mathrm{d}\bar\theta\mathcal{T}_p\mathrm{d}\theta\,,
\end{aligned} \tag{2.13}$$

where

$$\mathcal{T}_B = \sum_{p=\mathrm{odd}} \mathcal{T}_p = \mathrm{e}^{\mathcal{F}} S_B\tau_1\,, \tag{2.14}$$

and

$$S_B(\psi) = \psi + \frac{1}{3!}\tau_3\psi^3 + \frac{1}{5!}\psi^5 + \frac{1}{7!}\tau_3\psi^7 + \cdots\,, \tag{2.15}$$

where ψ is defined in (2.7). The supersymmetry transformations are given by

$$\begin{aligned}
\delta_\epsilon\theta &= \epsilon, \qquad \delta_\epsilon X^m = -i\bar\epsilon\Gamma^m\theta\,, \\
\delta_\epsilon(2\pi\alpha' A) &= -i\bar\epsilon\tau_3\Gamma^m\theta\mathrm{d}X^m + \frac{1}{6}\left(\bar\epsilon\tau_3\Gamma_m\theta\bar\theta\Gamma^m\mathrm{d}\theta + \bar\epsilon\Gamma_m\theta\bar\theta\tau_3\Gamma^m\mathrm{d}\theta\right).
\end{aligned} \tag{2.16}$$

The kappa transformations are

$$\begin{aligned}
\delta_\kappa\bar\theta &= \frac{1}{2}\bar\kappa\left(1+\Gamma_\kappa\right), \qquad \delta_\kappa X^m = -i\bar\theta\Gamma^m\delta_\kappa\theta\,, \\
\delta_\kappa(2\pi\alpha' A) &= i\delta_\kappa\bar\theta\tau_3\Gamma_m\theta\Pi^m + \frac{1}{2}\delta_\kappa\bar\theta\tau_3\Gamma_m\theta\bar\theta\Gamma^m\mathrm{d}\theta - \frac{1}{2}\delta_\kappa\bar\theta\Gamma_m\theta\bar\theta\tau_3\Gamma^m\mathrm{d}\theta\,,
\end{aligned} \tag{2.17}$$

where

$$\Gamma_\kappa = \frac{1}{(p+1)!}\frac{\varepsilon^{i_0\ldots i_p}}{\sqrt{-\det(G+\mathcal{F})}}\left(\rho_{p+1}\right)_{i_0\ldots i_p}\,, \tag{2.18}$$

and ρ_{p+1} is defined as a $(p+1)$-form given by

$$\rho_B = \sum_{p=\mathrm{odd}}\rho_{p+1} = \mathrm{e}^{\mathcal{F}}C_B(\psi)\tau_1\,, \tag{2.19}$$

where C_B is

$$C_B(\psi) = \tau_3 + \frac{1}{2!}\psi^2 + \frac{1}{4!}\tau_3\psi^4 + \frac{1}{6!}\psi^6 + \cdots\,. \tag{2.20}$$

We can switch on one more coupling in the world-volume consistent with all the symmetries of the Dp-brane action. From the spacetime point of view, it corresponds to turning on a closed $(p+1)$ RR field, which does not modify the flat supergravity equations of motion.

$$\mathcal{L} = -T_p\left[\mathcal{L}_{\mathrm{NG}} - \mathcal{L}_{\mathrm{WZ}} - \mathcal{L}_{C_{p+1}}\right], \tag{2.21}$$

where $\mathcal{L}_{C_{p+1}} = f^*C_{p+1}$ is the pullback of C_{p+1} on the world-volume (for more details see below).

3. Non-relativistic Dp branes

In this section we derive the action for non-relativistic Dp branes. The non-relativistic limit of strings [1, 2, 4] is obtained by decoupling some charged light degrees of freedom that obey a non-relativistic dispersion relation from the full relativistic theory. This is achieved by rescaling the world-volume fields with a dimensionless parameter ω and later sending the parameter to infinity. This limit implies that the transverse oscillations are small. For the case of Dp branes we should do the following rescaling

$$\begin{aligned} X^\mu &= \omega x^\mu\,, \\ X^a &= X^a\,, \\ T_p &= \omega^{1-p}T_{\mathrm{NR}}\,, \\ (2\pi\alpha')F_{ij} &= \omega f_{ij}\,, \\ (2\pi\alpha')A_i &= \omega W_i \\ \theta &= \sqrt{\omega}\theta_- + \frac{1}{\sqrt{\omega}}\theta_+\,, \\ C_{\mu_0\cdots\mu_p} &= -\varepsilon_{\mu_0\cdots\mu_p}\,, \end{aligned} \tag{3.1}$$

where X^m has been split in X^μ and X^a. The X^μ are the coordinates of target space parallel to the brane and X^a are the transverse coordinates. The NR gauge field strength is $f_{ij} = \partial_i W_j - \partial_j W_i$. The scaling of the fermions depends on the splitting of the fermions due the matrix Γ_*:

$$\Gamma_*\theta_\pm = \pm\theta_\pm\,. \tag{3.2}$$

The expression for Γ_* is

$$\Gamma_* = (-)^{n+1}\Gamma_{0\ldots p}\Gamma_{11}^{n+1}\,, \qquad p = 2n\,, \tag{3.3}$$

for type-IIA Dp branes and

$$\Gamma_* = \Gamma_{0\ldots p} i\tau_3^n \tau_2\,, \qquad p = 2n-1\,, \tag{3.4}$$

for type-IIB Dp branes. $\tau_{1,2,3}$ are the Pauli matrices. Γ_* appears as the first term of the non-relativistic expansion of the matrix Γ_κ appearing in the kappa transformations as will be shown below. Properties of the projected spinors are given in the appendix.

In order to compute the non-relativistic limit we should see how the forms involved in the action rescale under (3.1). The supertranslation 1-form (2.2) scales as

$$\Pi^\mu = \omega \hat{e}^\mu + \frac{i}{\omega}\bar{\theta}_+\Gamma^\mu \mathrm{d}\theta_+\,, \qquad \Pi^b = u^b\,, \tag{3.5}$$

where we have introduced

$$\begin{aligned} \hat{e}^\mu &= e^\mu + i\,\bar{\theta}_-\Gamma^\mu \mathrm{d}\theta_-, & e^\mu &= \mathrm{d}x^\mu\,,\\ u^a &= \mathrm{d}x^a + 2i\bar{\theta}_+\Gamma^a\mathrm{d}\theta_-\,, & x^a &= X^a + i\,\bar{\theta}_-\Gamma^a\theta_+. \end{aligned} \tag{3.6}$$

The form $\mathcal{F}$ scales as

$$\mathcal{F} = \omega\mathcal{F}^{(1)} + \frac{1}{\omega}\mathcal{F}^{(-1)}\,, \tag{3.7}$$

where for IIA

$$\begin{aligned} \mathcal{F}^{(1)} &= f + \Big[\left(i\bar{\theta}_-\Gamma_\mu\Gamma_{11}\mathrm{d}\theta_+ + i\bar{\theta}_+\Gamma_\mu\Gamma_{11}\mathrm{d}\theta_-\right)\left(\hat{e}^\mu - \frac{i}{2}\bar{\theta}_-\Gamma^\mu\mathrm{d}\theta_-\right) + \\ &\qquad + i\bar{\theta}_-\Gamma_a\Gamma_{11}\mathrm{d}\theta_-\left(u^a - \frac{i}{2}\bar{\theta}_-\Gamma^a\mathrm{d}\theta_+ - \frac{i}{2}\bar{\theta}_+\Gamma^a\mathrm{d}\theta_-\right)\Big] \qquad (3.8)\\ \mathcal{F}^{(-1)} &= \frac{1}{2}\left(\bar{\theta}_-\Gamma_\mu\Gamma_{11}\mathrm{d}\theta_+ + \bar{\theta}_+\Gamma_\mu\Gamma_{11}\mathrm{d}\theta_-\right)\bar{\theta}_+\Gamma^\mu\mathrm{d}\theta_+ + \\ &\qquad + i\bar{\theta}_+\Gamma_a\Gamma_{11}\mathrm{d}\theta_+\left(u^a - \frac{i}{2}\bar{\theta}_-\Gamma^a\mathrm{d}\theta_+ - \frac{i}{2}\bar{\theta}_+\Gamma^a\mathrm{d}\theta_-\right). \qquad (3.9) \end{aligned}$$

In order to have the expressions for IIB, we should replace Γ_{11} by τ_3.

Throughout the analysis, we keep ω large but finite in the intermediate computations and only send ω to infinity at the end. Therefore, we keep explicitly terms in the action that scale as positive powers of ω (which look superficially divergent) and terms that are independent of ω (which are finite). We drop terms that scale as inverse powers of ω because they cannot contribute when taking the limit at the end of the analysis.

The NG part of the (2.1) becomes after the rescalings

$$\begin{aligned} T_p\mathcal{L}_{\mathrm{NG}} &= T_p\sqrt{-\det\left(G_{ij}+\mathcal{F}_{ij}\right)}\\ &= T_{\mathrm{NR}}\omega^2\mathcal{L}_{\mathrm{NG}}^{\mathrm{div}} + T_{\mathrm{NR}}\mathcal{L}_{\mathrm{NG}}^{\mathrm{fin}} + \mathcal{O}(\omega^{-2}). \end{aligned} \tag{3.10}$$

The finite contribution is given by

$$\mathcal{L}_{\mathrm{NG}}^{\mathrm{fin}} = \frac{1}{2}\hat{e}\hat{g}^{jl}\vec{u}_l\vec{u}_j + i\hat{e}\bar{\theta}_+\hat{\gamma}^k\partial_k\theta_+ + \frac{1}{4}\hat{e}\mathcal{F}^{(1)}_{ij}\mathcal{F}^{(1)}_{k\ell}\hat{g}^{ik}\hat{g}^{j\ell}\,, \tag{3.11}$$

where $\hat{g}_{jk} = \eta_{\mu\nu}\hat{e}^\mu_j\hat{e}^\nu_k$, $\hat{e} = \det\hat{e}^\mu_j$ and $\hat{\gamma}_j = \hat{e}^\mu_j\Gamma_\mu$. We use the vector signs to indicate sums over the transverse space components. The superficially divergent contribution, written as a form, is given by

$$\mathrm{d}^{p+1}\sigma\mathcal{L}^{\mathrm{div}}_{\mathrm{NG}} = \hat{e}^0\cdots\hat{e}^p = -\frac{1}{(p+1)!}\varepsilon_{\mu_0\ldots\mu_p}\hat{e}^{\mu_0}\cdots\hat{e}^{\mu_p}\,. \tag{3.12}$$

Now we consider the scaling of the WZ term. For the IIA case we have

$$T_p h_{p+2} = T_{\mathrm{NR}}\omega^2 h^{(2)}_{p+2} + T_{\mathrm{NR}} h^{(0)}_{p+2} + \mathcal{O}(\omega^{-2})\,. \tag{3.13}$$

First, we analyse the superficially divergent term. This term comes from the expansion of the term in h_{p+2} that contains ψ to the power p.

$$h^{(2)}_{p+2} = -i\mathrm{d}\bar{\theta}_-\frac{1}{p!}\hat{e}^{\mu_1}\ldots\hat{e}^{\mu_p}\varepsilon_{\mu_1\ldots\mu_p\nu}\Gamma^\nu\mathrm{d}\theta_- = -\varepsilon_{\nu\mu_1\ldots\mu_p}\frac{1}{p!}\mathrm{d}\hat{e}^\nu\hat{e}^{\mu_1}\ldots\hat{e}^{\mu_p}\,. \tag{3.14}$$

We note that

$$\mathrm{d}(\mathrm{d}^{p+1}\sigma\mathcal{L}^{\mathrm{div}}_{\mathrm{GS}}) = \mathrm{d}(\mathrm{d}^{p+1}\sigma\mathcal{L}^{\mathrm{div}}_{\mathrm{NG}}) - h^{(2)}_{p+2} = 0\,. \tag{3.15}$$

As the last term involves only terms with fermions, this cancellation removes the terms with fermions in $\mathcal{L}^{\mathrm{div}}_{\mathrm{NG}}$. There remains the purely bosonic term in $\mathcal{L}^{\mathrm{div}}_{\mathrm{NG}}$, which is $e^0\cdots e^p$. Therefore, the potentially divergent term of $\mathcal{L}^{\mathrm{div}}_{\mathrm{GS}}$ is $e^0\cdots e^p$, which is a total derivative. This term can be cancelled by turning on a closed RR C_{p+1} form, given in (3.1), which only leads to the following potentially divergent term

$$\mathcal{L}^{\mathrm{div}}_{C_{p+1}} = -\frac{1}{(p+1)!}\varepsilon_{\mu_0\cdots\mu_p}e^{\mu_0}_0\cdots e^{\mu_p}_p\,. \tag{3.16}$$

Note that all the positively charged states are light. All states with non-positive charges become infinitely heavy and decouple.

The finite part of the action of a NR Dp brane is

$$S_{\mathrm{NR}} = -T_{\mathrm{NR}}\int\mathrm{d}\sigma^{p+1}\left(i\hat{e}\bar{\theta}_+\hat{\gamma}^k\partial_k\theta_+ + \frac{1}{2}\hat{e}\hat{g}^{jl}\vec{u}_l\vec{u}_j + \frac{1}{4}\hat{e}\mathcal{F}^{(1)}_{ij}\mathcal{F}^{(1)}_{k\ell}\hat{g}^{ik}\hat{g}^{j\ell}\right) + T_{\mathrm{NR}}\int\Omega^{(0)}_{p+1}\,, \tag{3.17}$$

where $\Omega^{(0)}_{p+1}$ is the non-relativistic WZ term. It has a complicated expression that we give below for the case of D1. In general, it verifies $\mathrm{d}\Omega^{(0)}_{p+1} = h^{(0)}_{p+2}$, where

$$\begin{aligned}\text{type IIA}:\ \ h^{(0)}_{p+2} &= (-)^n i\frac{1}{p!}\mathrm{d}\bar{\theta}_+\Gamma^{(n+1)}_{11}\hat{e}^{\mu_1}\ldots\hat{e}^{\mu_p}\Gamma_{\mu_1\ldots\mu_p}\mathrm{d}\theta_+ + \cdots\,,\\ \text{type IIB}:\ \ h^{(0)}_{p+2} &= i\frac{1}{p!}\mathrm{d}\bar{\theta}_+\left(\tau_3\right)^n i\tau_2\hat{e}^{\mu_1}\ldots\hat{e}^{\mu_p}\Gamma_{\mu_1\ldots\mu_p}\mathrm{d}\theta_+ + \cdots\,,\end{aligned} \tag{3.18}$$

and the dots indicate terms with dependence on θ_-.

3.1 Supersymmetry and kappa transformations

The relativistic Dp-brane action (2.1) is invariant under the supersymmetry (2.8) and kappa transformations (2.9) for type IIA and (2.16) and (2.17), respectively, for type IIB. In order to obtain the non-relativistic counterpart of these transformations that leave the NR Dp-brane action, (3.17), invariant, we should rescale the supersymmetry parameter

$$\epsilon = \sqrt{\omega}\epsilon_- + \sqrt{\frac{1}{\omega}}\epsilon_+ \,, \tag{3.19}$$

and the kappa parameter

$$\kappa = \sqrt{\omega}\kappa_- + \sqrt{\frac{1}{\omega}}\kappa_+ \,. \tag{3.20}$$

We also need the expansion of the kappa gamma matrix,

$$\Gamma_\kappa = \Gamma_* + \frac{1}{\omega}\Gamma_\bullet + \mathcal{O}(\omega^{-2}) \,, \tag{3.21}$$

where Γ_* was introduced before in (3.3) and, for type IIA,

$$\Gamma_\bullet = -\hat{\gamma}^k \Gamma_a u_k^a \Gamma_* - \frac{1}{2}\mathcal{F}^{(1)}_{jk}\Gamma_{11}\hat{\gamma}^j\hat{\gamma}^k\Gamma_* \,. \tag{3.22}$$

The symmetries of the non-relativistic lagrangian are a consequence of the symmetries of the parent relativistic theory and the fact that the divergent term of the non-relativistic expansion, $\mathcal{L}^{\rm div}_{\rm GS} = \mathcal{L}^{\rm div}_{\rm NG} - \mathcal{L}^{\rm div}_{\rm WZ}$, is a total derivative or is absent when we introduce the coupling to the RR C_{p+1} form (3.16).

The supersymmetry transformations of the NR Dp-brane action for type IIA (3.17) are given by

$$\begin{aligned}
\delta_\epsilon\theta_- &= \epsilon_- \,, \qquad \delta_\epsilon\theta_+ = \epsilon_+ \,, \\
\delta_\epsilon x^\mu &= i\bar{\theta}_-\Gamma^\mu\epsilon_- \,, \qquad \delta_\epsilon X^a = i\bar{\theta}_-\Gamma^a\epsilon_+ + i\bar{\theta}_+\Gamma^a\epsilon_- \,, \qquad \delta_\epsilon x^a = 2i\bar{\theta}_-\Gamma^a\epsilon_+ \,, \\
\delta_\epsilon W &= -i(\bar{\epsilon}_+\Gamma_\mu\Gamma_{11}\theta_- + \bar{\epsilon}_-\Gamma_\mu\Gamma_{11}\theta_+)\mathrm{d}x^\mu - i\bar{\epsilon}_-\Gamma_a\Gamma_{11}\theta_-\mathrm{d}X^a + \\
&\quad + \frac{1}{6}\Big[(\bar{\epsilon}_+\Gamma_\mu\Gamma_{11}\theta_- + \bar{\epsilon}_-\Gamma_\mu\Gamma_{11}\theta_+)\bar{\theta}_-\Gamma^\mu\mathrm{d}\theta_- + \\
&\qquad + \bar{\epsilon}_-\Gamma_a\Gamma_{11}\theta_-\left(\bar{\theta}_-\Gamma^m\mathrm{d}\theta_+ + \bar{\theta}_+\Gamma^m\mathrm{d}\theta_-\right) + \\
&\qquad + \bar{\epsilon}_-\Gamma_\mu\theta_-(\bar{\theta}_-\Gamma^\mu\Gamma_{11}\mathrm{d}\theta_+ + \bar{\theta}_+\Gamma^\mu\Gamma_{11}\mathrm{d}\theta_-) + \\
&\qquad + (\bar{\epsilon}_-\Gamma_a\theta_+ + \bar{\epsilon}_+\Gamma_a\theta_-)\bar{\theta}_-\Gamma^m\Gamma_{11}\mathrm{d}\theta_-\Big] \,.
\end{aligned} \tag{3.23}$$

The action (3.17) has also the NR kappa symmetry

$$\begin{aligned}
\delta_\kappa\bar{\theta}_- &= \bar{\kappa}_- \,, \qquad \delta_\kappa\bar{\theta}_+ = (-)^n\frac{1}{2}\bar{\kappa}_-\Gamma_\bullet, \\
\delta_\kappa x^\mu &= -i\bar{\theta}_-\Gamma^\mu\kappa_- \,, \\
\delta_\kappa X^a &= -i\bar{\theta}_+\Gamma^a\kappa_- + (-)^n i\bar{\kappa}_-\frac{\Gamma_\bullet}{2}\Gamma^a\theta_-, \qquad \delta_\kappa x^a = -2i\bar{\theta}_+\Gamma^a\kappa_- \,, \\
\delta_\kappa W &= i(\delta_\kappa\bar{\theta}_+\Gamma_\mu\Gamma_{11}\theta_- + \delta_\kappa\bar{\theta}_-\Gamma_\mu\Gamma_{11}\theta_+)\hat{e}^\mu + i\delta_\kappa\bar{\theta}_-\Gamma^a\Gamma_{11}\theta_-u^a +
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}(\delta_\kappa\bar\theta_+\Gamma_\mu\Gamma_{11}\theta_- + \delta_\kappa\bar\theta_-\Gamma_\mu\Gamma_{11}\theta_+)\bar\theta_-\Gamma^\mu \mathrm{d}\theta_- + \\
&+\frac{1}{2}\delta_\kappa\bar\theta_-\Gamma_a\Gamma_{11}\theta_-(\bar\theta_+\Gamma^a\mathrm{d}\theta_- + \bar\theta_-\Gamma^a\mathrm{d}\theta_+) - \\
&-\frac{1}{2}\delta_\kappa\bar\theta_-\Gamma_\mu\theta_-(\bar\theta_-\Gamma^\mu\Gamma_{11}\mathrm{d}\theta_+ + \bar\theta_+\Gamma^\mu\Gamma_{11}\mathrm{d}\theta_-) - \\
&-\frac{1}{2}(\delta_\kappa\bar\theta_-\Gamma_a\theta_+ + \delta_\kappa\bar\theta_+\Gamma_a\theta_-)\bar\theta_-\Gamma^a\Gamma_{11}\mathrm{d}\theta_- \,. \qquad (3.24)
\end{aligned}
$$

From (3.24) we see that θ_- is a gauge degree of freedom that can be eliminated by choosing $\theta_- = 0$. In this gauge we can explicitly integrate the WZ term. The action for a non-relativistic Dp brane in diffeomorphism-invariant form becomes

$$
\begin{aligned}
S_{\rm NR} = -T_{\rm NR}\int \mathrm{d}\sigma^{p+1}\Bigg[&\frac{1}{2}\sqrt{-\det g}\, g^{ij}\partial_i\vec X\partial_j\vec X + 2i\sqrt{-\det g}\,\bar\theta_+\gamma^i\partial_i\theta_+ + \\
&+\frac{1}{4}\sqrt{-\det g}\, f_{ij}f_{k\ell}g^{ik}g^{j\ell}\Bigg]\,. \qquad (3.25)
\end{aligned}
$$

where g^{ij} is the inverse of the induced metric $g_{ij} = \eta_{\mu\nu}e^\mu_j e^\nu_j$. This lagrangian is interacting since the longitudinal scalars $x^\mu(\sigma)$ are coupled to the transverse scalars $X^a(\sigma)$ and the dynamical fermions $\theta_+(\sigma)$ via the induced metric g_{ij}. The gamma matrices γ_i are the pullbacks of the gamma matrices in spacetime, $\gamma_i = e^\mu_i\Gamma_\mu$.

In the static gauge ($x^\mu = \sigma^\mu$), this theory becomes a supersymmetric free theory in a flat spacetime of scalars, fermions and gauge fields.

$$
S_{\rm NR} = -T_{\rm NR}\int \mathrm{d}^{p+1}\sigma\left[\frac{1}{2}\eta^{ij}\partial_i\vec X\partial_j\vec X + 2i\bar\theta_+\Gamma^i\partial_i\theta_+ + \frac{1}{4}f_{ij}f_{k\ell}\eta^{ik}\eta^{j\ell}\right]. \qquad (3.26)
$$

Once kappa symmetry is fixed, sixteen of the supersymmetries are linearly realized while the other sixteen are non-linearly realized. The non-linear realized supersymmetries are generated by ϵ_+, while the linearly realized supersymmetries are induced by ϵ_-. The transformations are

$$
\begin{aligned}
\delta\bar\theta_+ &= \bar\epsilon_+ - \frac{1}{2}\bar\epsilon_-\left(\Gamma^k\partial_k X^a\Gamma_a + \frac{1}{2}f_{jk}\Gamma_{11}\Gamma^{jk}\right), \\
\delta X^a &= 2i\bar\theta_+\Gamma^a\epsilon_-\,, \\
\delta W_i &= -2i\bar\epsilon_-\Gamma_i\Gamma_{11}\theta_+\,. \qquad (3.27)
\end{aligned}
$$

For type-IIB Dp branes we obtain the same expressions as for IIA but with the substitution of Γ_{11} by τ_3, at this point we should note that the substitution must be done before any commutation of Γ_{11} with any other Γ. The only exception is the kappa symmetry transformation for the spinor θ_+ (3.24), which is written for the IIB case as

$$
\delta_\kappa\bar\theta_+ = \frac{1}{2}\bar\kappa_-\Gamma_\bullet\,. \qquad (3.28)
$$

Consequently, the residual transformation is

$$
\begin{aligned}
\delta\bar\theta_+ &= \bar\epsilon_+ - \frac{1}{2}\bar\epsilon_-\left(\Gamma^k\partial_k X^a\Gamma_a + \frac{1}{2}f_{jk}\tau_3\Gamma^{jk}\right), \\
\delta X^a &= 2i\bar\theta_+\Gamma^a\epsilon_-\,, \\
\delta W_i &= -2i\bar\epsilon_-\Gamma_i\tau_3\theta_+\,. \qquad (3.29)
\end{aligned}
$$

The linearly realized supersymmetries represent the transformations of a vector multiplet with 16 real supersymmetries in $p+1$ dimensions. The formulae (3.27) and (3.29) give a uniform way for writing these vector multiplet transformations in any dimension using $D=10$ notation.

3.2 D1 string

In this section we will consider explicitly the case of D1 strings. This case is interesting because we can write explicitly the non-relativistic Wess-Zumino term, since we can easily integrate the h_3 form given by (3.18), and therefore we can write explicitly the kappa-symmetric form of the non-relativistic D1 string action.

For the D-string we have $\Gamma_* = \Gamma_0\Gamma_1\tau_1$. As in the general case, we obtain a divergent term for the WZ part, which we can write explicitly as

$$\mathcal{L}^{\rm div}_{WZ} = \varepsilon^{jk}\varepsilon_{\mu\nu} i\bar{\theta}_-\Gamma^\nu\partial_j\theta_-\left(\partial_k x^\mu + \frac{i}{2}\bar{\theta}_-\Gamma^\mu\partial_k\theta_-\right). \tag{3.30}$$

The ω^2 terms of $\mathcal{L}_{\rm GS}$ (remember that $\mathcal{L}_{\rm GS} = \mathcal{L}_{\rm NG} - \mathcal{L}_{\rm WZ}$) give

$$\mathcal{L}^{\rm div}_{\rm GS} = -\frac{1}{2}\varepsilon^{jk}\varepsilon_{\mu\nu}\partial_j x^\mu\partial_k x^\nu\,. \tag{3.31}$$

This divergent term can be cancelled by turning on a closed RR C_2 form (3.16).

The finite part of the kappa-symmetric form of the action is

$$\begin{aligned} S_{\rm NR} = -T_{\rm NR}\int {\rm d}^2\sigma\Big[&2\hat{e}i\bar{\theta}_+\hat{\gamma}^k\partial_k\theta_+ + \frac{1}{2}\hat{e}\hat{g}^{lk}\vec{u}_l\vec{u}_k + \\ &-\frac{1}{4}\hat{e}\hat{g}^{kl}\mathcal{F}^{(1)}_{lj}\hat{g}^{ji}\mathcal{F}^{(1)}_{ik} + 2i\varepsilon^{jk}\bar{\theta}_+\Gamma_a\tau_1\partial_j\theta_-(u^a_k - i\bar{\theta}_+\Gamma^a\partial_k\theta_-)\Big]. \end{aligned} \tag{3.32}$$

If we choose $\theta_- = 0$ and the static gauge, the action becomes

$$S_{\rm NR} = -T_{\rm NR}\int {\rm d}^2\sigma\left[\frac{1}{2}\eta^{ij}\partial_i\vec{X}\partial_j\vec{X} + 2i\bar{\theta}_+\Gamma^i\partial_i\theta_+ + \frac{1}{4}f_{ij}f_{k\ell}\eta^{ik}\eta^{j\ell}\right]. \tag{3.33}$$

The residual supersymmetry transformation is given by (3.29) for $p=1$.

4. Conclusions

Non-relativistic superstrings and Dp branes describe a consistent and soluble sector of the full relativistic string theory. In this paper, we derived the world-volume theory of non-relativistic supersymmetric Dp branes in flat spacetime. This is achieved by considering a suitable non-relativistic limit of relativistic wound Dp branes. The branes are charged with respect to the C_{p+1} RR form, and we fine-tune the coupling in such a way that the tension of the Dp-brane in cancelled by the RR coupling. This is the cancellation of the superficially divergent terms in the action. It is important to notice that kappa symmetry is crucial for this cancellation. The balance between the NG part and the WZ part of the action needed for kappa symmetry is the same balance that is necessary for combining these superficially divergent terms in a total derivative. Then this total derivative can be cancelled by a closed C_{p+1} form.

Once all the gauge symmetries of the non-relativistic Dp-brane action are fixed, the world-volume theory reduces to a supersymmetric field theory of bosons, fermions and gauge fields in flat spacetime.

The non-relativistic string theory provides a new soluble sector of string theory where one could test the gauge/gravity correspondence. See [6] for a concrete proposal in the case of $AdS_5 \times S^5$. More in general, it could be interesting to study the non-relativistic sector of AdS branes, e.g. [14].

Acknowledgments

We are grateful to Jaume Gomis, Kiyoshi Kamimura and Paul Townsend for interesting and very useful discussions.

This work is supported in part by the European Community's Human Potential Programme under contract MRTN-CT-2004-005104 'Constituents, fundamental forces and symmetries of the universe'. The work is supported in part by the FWO - Vlaanderen, project G.0235.05 and by the Federal Office for Scientific, Technical and Cultural Affairs through the "Interuniversity Attraction Poles Programme — Belgian Science Policy" P5/27, by the Spanish grant MYCY FPA 2004-04582-C02-01, and by the Catalan grant CIRIT GC 2001, SGR-00065.

Joaquim Gomis acknowledges the Francqui Foundation for the Interuniversity International Francqui chair in Belgium awarded to him, as well the warm hospitality at the University of Leuven.

A. Notation and some useful formulae

Here we summarize our notation. Indices are

$$\begin{aligned} \text{target space}: &\quad m,n = 0,\ldots,9 \\ \text{target space, longitudinal}: &\quad \mu,\nu = 0,\ldots,p\,, \\ \text{target space, transverse}: &\quad a,b = p+1,\ldots,9\,, \\ \text{world}-\text{volume}: &\quad i,j = 0,\ldots,p\,. \end{aligned} \tag{A.1}$$

The metric in target space and on the world-volume has signature mostly +. The totally antisymmetric Levi-Civita tensor is normalized by $\varepsilon^{012\ldots p} = +1$, $\varepsilon_{012\ldots p} = -1$.

The Γ^m and Γ_{11} can be chosen real by taking the charge conjugation matrix $C = \Gamma_0$, and

$$\Gamma_{11} = \Gamma_0\Gamma_1\ldots\Gamma_9\,. \tag{A.2}$$

For type-IIA theories, θ is a Majorana spinor, while for type-IIB theories, there are two Majorana-Weyl spinors θ_α ($\alpha = 1,2$) of the same chirality. The index α is not displayed explicitly. The Pauli matrices τ_1, τ_2, τ_3 act on it. This leads to some useful symmetry relations as

$$\bar\chi\lambda = \bar\lambda\chi\,, \qquad \lambda = \Gamma_m\epsilon \to \bar\lambda = -\bar\epsilon\Gamma_m\,, \qquad \lambda = \Gamma_{11}\epsilon \to \bar\lambda = -\bar\epsilon\Gamma_{11}\,. \tag{A.3}$$

There are cyclic identities

$$\sum_{IJK\text{ cyclic}} \left[\Gamma_m\theta_I\,(\bar\theta_J\Gamma^m\,\theta_K) + \Gamma_m\Gamma_{11}\theta_I\,(\bar\theta_J\Gamma^m\Gamma_{11}\,\theta_K)\right] = 0\,, \tag{A.4}$$

and, for type-IIB spinors,

$$\sum_{I\ J\ K\text{ cyclic}} \left\{\Gamma_m\tau_1\theta_I\left(\bar\theta_J\Gamma^m\theta_K\right) + \Gamma_m\theta_I\left(\bar\theta_J\Gamma^m\tau_1\theta_K\right)\right\} = 0\,, \tag{A.5}$$

where τ_1 can also be replaced by τ_3.

We define projections in (3.2), using the matrix Γ_* defined in (3.3) and in (3.4) for type IIA and IIB, respectively. This matrix squares to $\mathbb{1}$. Here are some useful properties:

$$\begin{aligned}
&\Gamma_*\theta_\pm = \pm\theta_\pm\,,\\
&\text{type IIA}: \quad \bar\theta_\pm = \pm(-)^{\frac{p}{2}+1}\bar\theta_\pm\Gamma_*\,, \qquad \Gamma_*\Gamma_{11} = -\Gamma_{11}\Gamma_*\,,\\
&\qquad\qquad \Gamma_*\Gamma^\mu = (-)^{\frac{p}{2}+1}\Gamma^\mu\Gamma_*\,, \qquad \Gamma_*\Gamma^a = (-)^{\frac{p}{2}}\Gamma^a\Gamma_*\,,\\
&\text{type IIB}: \quad \bar\theta_\pm = \mp\bar\theta_\pm\Gamma_*\,, \qquad \Gamma_*\tau_3 = -\tau_3\Gamma_*\,,\\
&\qquad\qquad \Gamma_*\Gamma^\mu = -\Gamma^\mu\Gamma_*\,, \qquad \Gamma_*\Gamma^a = \Gamma^a\Gamma_*\,.
\end{aligned} \tag{A.6}$$

For the D1 string we can also use

$$\Gamma_\mu\tau_1\theta_\pm = \pm\varepsilon_{\mu\nu}\Gamma^\nu\theta_\pm. \tag{A.7}$$

Differently from [8, 7] the differentials and the spinors have independent gradings. Components of the forms are defined by

$$A_r = \frac{1}{r!}A_{i_1\ldots i_r}\mathrm{d}\sigma^{i_1}\ldots\mathrm{d}\sigma^{i_r}\,, \tag{A.8}$$

and differentials are taken from the left.

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